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Electroweak effects in radiative B decays

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Abstract: We compute the two-loop electroweak corrections to the radiative decays of the B meson in the SM. Electroweak effects reduce the Wilson coefficient $C_7^{\rm eff}(M_W)$ by 2.6% for a light Higgs boson of about 100 GeV and are less important for a heavier Higgs. The leading term of a heavy top expansion of our result differs from the one obtained in the *gaugeless* approximation where only top quark Yukawa couplings are considered: we discuss the origin of the discrepancy and provide a criterion for the validity of the *gaugeless* approximation. As a byproduct of the calculation we also obtain the $\mathrm{O}(\alpha)$ corrections to the Wilson coefficient of the four-fermion operator Q_2. A careful analysis of the interplay between electroweak and QCD effects leads to an overall 2% reduction of the total branching ratio for $B \to X_s \gamma$ due to purely electroweak corrections. For a light Higgs boson, the up-to-date SM prediction is $BR_\gamma = (3.29 \pm 0.21 \pm 0.21) \times 10^{-4}$.

Keywords: Standard Model, Rare Decays, B-Physics.

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Contents

1. Introduction

Radiative B decays represent one of the most important probes of new physics and a major testing ground for the Standard Model (SM). They already place severe constraints on many new physics scenarios. The present experimental accuracy for the branching ratio of $B \to X_s\gamma$ (BR_γ in the following) is less than 15% [1] and is expected to improve significantly in the near future, both at CLEO and at the B factories.

On the theoretical side, since precise predictions in the SM are particularly important, the subject has reached a high degree of technical sophistication. Indeed, perturbative QCD corrections are very sizeable [2] and give the dominant contribution; they are best implemented in the framework of an effective theory obtained by integrating out the heavy degrees of freedom characterized by a mass scale $M \geq M_W$. At lowest order in this approach the FCNC processes $B \to X_s\gamma$ and $B \to X_s g$ proceed through helicity violating amplitudes induced by the magnetic operators

$$Q_7 = \frac{e}{4\pi^2} m_b\, \bar{s}_L \sigma^{\mu\nu} b_R\, F_{\mu\nu}\,, \qquad Q_8 = \frac{g_s}{4\pi^2} m_b\, \bar{s}_L \sigma^{\mu\nu} t^a b_R\, G^a_{\mu\nu}\,. \tag{1.1}$$

A few years ago the renormalization group improved QCD calculation has been completed at the next-to-leading order (NLO) [3]–[6], reducing the uncertainty from uncalculated QCD higher orders to about 5%. More recently, NLO predictions have been made available in some new physics models as well [5, 7].

There has also been progress concerning QED and electroweak radiative corrections: after Czarnecki and Marciano considered all the leading QED logarithms [8], their interplay with QCD corrections has been studied in [9, 10], and Strumia [11] has calculated the leading term of the Heavy Top Expansion (HTE) of the electroweak two-loop corrections using the *gaugeless* limit of the SM. As for non-perturbative effects, they seem to be under control [12], although some aspects may still need a more detailed investigation [13].

In addition to uncalculated radiative corrections and non-perturbative effects, the calculation of BR_γ is also affected by the uncertainties on the input parameters (the CKM matrix elements, the semileptonic branching ratio $\mathrm{BR}_{\mathrm{SL}}$, etc.). In fact, the latter bring the overall theoretical error to about 10%. As the parametric uncertainties (especially those on the CKM elements, α_s, $\mathrm{BR}_{\mathrm{SL}}$, and M_t) are expected to decrease soon, and given the crucial importance of this decay mode, it seems appropriate to try and refine the SM prediction as much as possible.

In this note we reexamine the two-loop electroweak contributions to radiative B decays and present the result of a calculation where only some photonic effects have been neglected. Moreover, we update the SM prediction of BR_γ using the latest experimental inputs. The final result is expressed by a compact formula that summarizes the dependence on the input parameters.

2. The calculation

Although generally small, two-loop purely electroweak effects are sometimes very important: an example is provided by the precision observables of the SM, like the effective sine measured on the Z^0 pole and the W mass, where radiative corrections up to $O(g^4 M_t^2/M_W^2)$ [14, 15] are now routinely included in the analysis with important consequences in the electroweak fits [15, 16]. Moreover, by fixing the normalization of the electroweak coupling, two-loop effects reduce the electroweak scheme dependence of the SM prediction, which can be quite large — also for FCNC processes [17].

As mentioned above, in the calculation of BR_γ the leading large logarithms of QED origin are now under control, as a resummation of all $\alpha\alpha_s^{n-1}(\ln m_b/M_W)^n$ terms has been completed [8, 9, 10]. Apart from that, our knowledge of electroweak effects in $b \to s\gamma$ is limited to the subset of two-loop fermion loop corrections calculated in [8] and to the leading term of the HTE of [11]. In fact, the two results are numerically very different — about -2.3% and less than -0.7%, respectively, on the Wilson coefficient at M_W. The leading term of the HTE was calculated in [11] using the *gaugeless* limit of the SM, i.e. in a Yukawa theory where the heavy top couples only to the Higgs doublet, setting $M_W = 0$ and keeping the Higgs mass M_H finite and arbitrary. In the presence of external gauge bosons, these can be considered as background sources. This approach presents a few limitations that also motivate our new calculation:

- the lowest order contribution to the Wilson coefficient of Q_7 is a function of the top mass whose HTE converges very slowly. Using $x_t = M_t^2/M_W^2 \approx 4.7$ and writing explicitly the numerical values of the successive $O(1/x_t^n)$ terms, it reads

$$
\begin{aligned}
C_7^{(0)}(x_t) &= \frac{x_t(7-5x_t-8x_t^2)}{24(x_t-1)^3} + \frac{x_t^2(3x_t-2)\ln x_t}{4(x_t-1)^4} \\
&= -\frac{1}{3} - 0.010 + 0.070 + 0.046 + 0.021 + \cdots = -0.195\,, \qquad (2.1)
\end{aligned}
$$

where the ellipses represent contributions $O(1/x_t^5)$ or higher. The leading HTE is therefore unlikely to provide anything more than an order of magnitude estimate of the two-loop electroweak contribution. In this respect, the similar case of $B_0-\bar{B}_0$ mixing [17] is very instructive: for realistic values of the top mass the complete two-loop electroweak correction is not well approximated even by the first three terms of the HTE and the leading HTE term is numerically far from the complete result.

- even assuming the leading HTE term to be representative, it should not be expected to give an accurate result for a light Higgs mass, $M_H \approx O(M_W)$, because it is obtained by setting $M_W = 0$ [15]. On the other hand, present electroweak fits show a decisive preference for a light Higgs boson, $M_H <$ 215 GeV at 95% C.L. [16].

- the *gaugeless* limit has often been used to compute the leading HTE term, but it is known [18] that in some cases it does *not* reproduce the correct result. In the following we explain why it fails for radiative B decays and provide a general criterion for its use.

A complete calculation of all electroweak effects in radiative B decays in the framework of effective hamiltonians is a very complex enterprise which involves other operators in addition to those of eq. (1.1). In fact, the analysis should be aimed at resumming all $\alpha\alpha_s^n\,(\ln m_b/M_W)^n$ effects. The procedure is summarized, for instance, in [19]. Its necessary steps would be:

(i) the calculation of two-loop O(α) matching conditions for $Q_{7,8}$ at some $O(M_W)$ scale — this involves also their QED mixing with all other operators — and of the O(α) contributions to the coefficients of various four quark operators;

(ii) QED-QCD running of the Wilson coefficients down to the B mass scale — this would require a three loop computation of the anomalous dimension matrix similar to that of [3];

(iii) calculation of the one-loop QED matrix elements of the various operators — the determination of these matrix elements depends sensitively on the precise experimental conditions.

An important simplification can be obtained by keeping only the first term in an expansion around $s_W = \sin\theta_W = 0$. This is equivalent to considering a $\mathrm{SU}(2)_L$ theory with a background photon field and removes all the light virtual degrees of freedom. In particular, all the diagrams with virtual photons and all infrared (IR) divergences drop out of the two-loop calculation in a gauge-invariant way. Step (i) is therefore much simpler as the calculation of the two-loop $b \to s\gamma$ and $b \to sg$ amplitudes gives us directly the scheme independent $O(g^2)$ correction to $C_{7,8}$, respectively ($g = e/s_W$ is the $\mathrm{SU}(2)_L$ coupling). Moreover, this simplification avoids completely steps (ii) and (iii), because they are both driven by purely photonic effects suppressed at least by $Q_u|Q_d|s_W^2 \approx 0.05$ with respect to pure $\mathrm{SU}(2)_L$ contributions.

In analogy to [19], we complement this approximation scheme by keeping also the $O(g^2 M_t^2/M_W^2)$ contributions that vanish as $s_W \to 0$. In practice, we therefore expand the two-loop $b \to s\gamma(g)$ amplitude $A^{(2)}$ in powers of s_W^2:

$$A^{(2)} = g^4 \left[A_0 + A_1 s_W^2 + O\left(s_W^4\right)\right] \tag{2.2}$$

and retain only A_0 and the $O(M_t^2)$ part of A_1. This is likely to be a sufficiently good approximation, as suggested by those cases [17, 20] where it has been possible to compare it with the complete SM result. We recall that $\Delta\rho$ [20] and $B_0 - \bar{B}_0$ mixing [17] involve amplitudes conceptually similar to those under consideration: forbidden at tree level and induced at one-loop by virtual $\mathrm{SU}(2)_L$ effects. Later on, we will give an estimate of the residual uncertainty.

We calculate analytically the two-loop amplitudes in the Feynman background gauge with a background photon (gluon). A few thousand diagrams are automatically generated by the package *FeynArts 2.2* [21] (the topologies are shown in figure 1). After setting to zero all light fermion masses but the b-quark mass, they can be reduced to a few hundred equivalence classes, which we have actually computed. Due to the GIM mechanism, the CKM cofactor of each equivalence class is always proportional to $\lambda_t = V_{tb}V_{ts}^*$. The extraction of the magnetic penguin amplitude and the two-loop integration are performed as in [5]. All the steps of the calculation have been implemented in two independent and completely automatic codes that involve various combinations of Mathematica [22] and Form [23] routines. Although the result can be expressed in terms of logarithms and dilogarithms, it is rather lengthy and we will present instead accurate numerical approximations.

A peculiarity of the two-loop calculation for these processes is the presence of diagrams containing anomalous fermionic loops (triangles). It is well known that the naive definition of anticommuting γ_5 in n dimensions that we employ in the rest of the calculation fails for these diagrams because it leads to algebraical ambiguities and cannot reproduce the axial anomaly. Our solution consists in calculating Dirac structures containing an odd number of γ_5's — i.e. those leading to the anomalous term — using anticommuting γ_5 in strictly four dimensions, which is possible because of their apparent UV convergence. The anomaly cancellation then guarantees the

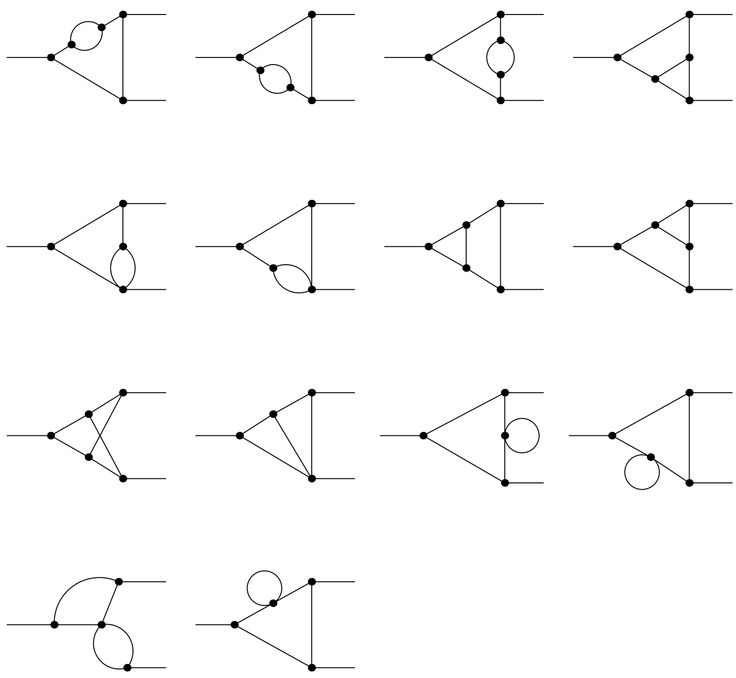

Figure 1: Two-loop topologies for $b \to s\gamma$.

absence of both anomalous and ambiguous terms in the sum of all diagrams (see [25, appendix C]). We have also checked our results for these specific terms using the HV definition of γ_5 in n dimensions [24]. From a formal point of view, the equivalence of the two methods follows from the absence of non-invariant counterterms for the odd γ_5 part in the HV case [25], as can be also seen in full generality using the powerful formalism of [26].

The renormalization is performed following the simple framework of [17], to which we refer for a detailed discussion (see also [15]) and for the notation. We recall that the top mass is renormalized on-shell as far as electroweak effects are concerned, but it is customary to use an $\overline{\mathrm{MS}}$ definition for the QCD effects. Although the choice of the scale μ_t for the $\overline{\mathrm{MS}}$ top mass is a matter of convention, the NLO QCD corrections depend sensitively on μ_t. For simplicity, we follow [5] and in the numerics we set $\mu_t = \mu_W = M_W$ and employ $\overline{M}_t \equiv \overline{M}_t(M_W) = 175.5 \pm 5.1\,\mathrm{GeV}$ obtained from the pole mass value $M_t^{\mathrm{pole}} = 174.3 \pm 5.1\,\mathrm{GeV}$ [27]. No renormalization of the electric charge is necessary in our approximation. The renormalization of the b-quark mass, not needed in [17], is also performed on-shell. The counterterm reads

$$\frac{\delta m_b}{m_b} = \frac{g^2}{16\pi^2}\left(\frac{\bar{\mu}^2}{\overline{M}_t^2}\right)^{\epsilon}\left[\frac{3}{8\epsilon}(x_t - 1) - \frac{3 + 8x_t - 5x_t^2}{16(x_t - 1)} - \frac{3\,(1 - 3x_t + x_t^2)\ln x_t}{8(x_t - 1)^2} + O(s_W^2)\right].$$

One should keep in mind that the m_b factors in eq. (1.1) originate either from the b-quark Yukawa coupling or from the use of on-shell equations of motion. In the latter case, m_b should *not* be renormalized as it is on-shell by definition. This is the mass appearing explicitly in the projector of [5, eq. (14)]. Indeed, besides the magnetic operators of eq. (1.1), there are additional off-shell operators that project onto $Q_{7,8}$ when the external momenta are set on-shell, i.e. $Q_{10} \sim e\,\bar{s}_L\{\not{D}, \sigma^{\mu\nu}\} b_L\, F_{\mu\nu}$ and the analogous one with gluon fields [5]. In correcting the external fields, one should take into account that the chirality of the b quark is different in Q_7 and Q_{10}. The external leg corrections of [17] correspond to the correct LSZ factors and implement the renormalization of the CKM matrix according to [28] within our approximations. We recall that this gauge invariant definition of the CKM matrix is the most appropriate to the present low-energy measurements because, unlike an $\overline{\mathrm{MS}}$ renormalization, it avoids $O(g^2)$ corrections not suppressed by GIM and proportional to $(m_i^2+m_j^2)/(m_i^2-m_j^2)$, where $m_{i,j}$ are light quark masses [28]. Notice also that in our framework $\delta(Z_d^R)_{ij}=0$.

According to the standard procedure, we will normalize BR_γ to the semileptonic branching ratio, $\mathrm{BR}_{\mathrm{SL}}$. This fixes the normalization of the electroweak coupling but requires the inclusion of the one-loop electroweak corrections to $\mathrm{BR}_{\mathrm{SL}}$ [29]. It is straightforward to see that, up to $O(s_W^2)$ terms that we neglect, these are the same that enter the muon decay. Hence, in this respect the use of $\mathrm{BR}_{\mathrm{SL}}$ is effectively equivalent to that of the Fermi constant measured in muon decays, G_μ, and the coupling renormalization proceeds as described for this case in [17]. We incorporate the complete one-loop correction to the muon decay amplitude, without taking the $s_W \to 0$ limit, on the ground that this is an independent process for which the complete correction is available. Notice that the leading part of the photonic corrections to $\mathrm{BR}_{\mathrm{SL}}$, characterized by large logarithms and not considered in our calculation is part of the $O(\alpha\alpha_s^{n-1}\ln^n m_b/M_W)$ analysis of [8, 9, 10] and is included in our numerical results.

We now recall that the regularization scheme-independent quantity entering the calculation of BR_γ is not $C_7(\mu_b)$ but a combination $C_7^{\mathrm{eff}}(\mu_b)$ of this Wilson coefficient and of the coefficients of the four fermion operators with mixed chirality [30, 3]. It turns out [10] that the two scheme-independent quantities relevant for $B \to X_s\gamma$ and $B \to X_s g$ are

$$
\begin{aligned}
C_7^{\mathrm{eff}}(\mu) &= C_7(\mu) - \frac{1}{3}C_5(\mu) - C_6(\mu) + \frac{1}{6}C_7^P(\mu) + \frac{1}{2}C_8^P(\mu)\,, \\
C_8^{\mathrm{eff}}(\mu) &= C_8(\mu) + C_5(\mu) - \frac{1}{2}C_7^P(\mu)\,,
\end{aligned}
\tag{2.3}
$$

where $C_{5,6}$ and $C_{7,8}^P$ are the Wilson coefficient of the four-quark operators

$$
\begin{aligned}
Q_5 &= (\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}q)_{V+A}\,, \qquad & Q_6 &= (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A}\,, \\
Q_7^P &= \frac{3}{2}(\bar{s}d)_{V-A} \sum_{q=u,d,s,c,b} e_q(\bar{q}q)_{V+A}\,, \qquad & Q_8^P &= \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s,c,b} e_q(\bar{q}_\beta q_\alpha)_{V+A}\,.
\end{aligned}
\tag{2.4}
$$

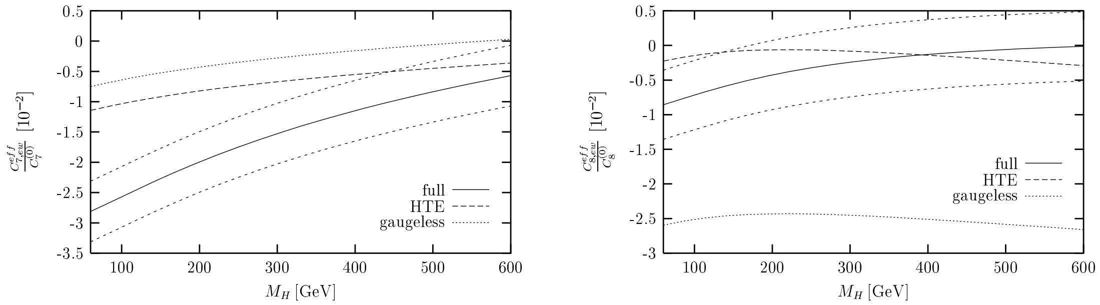

Figure 2: Electroweak corrections to the Wilson coefficients $C_{7,8}(M_W)$. The solid lines represent our results with their error estimates, the dashed lines their leading HTE, and the dotted lines the results of the *gaugeless* approximation.

As $C_8^P(M_W)$ and $C_{5,6}(M_W)$ do not receive any $\mathrm{O}(\alpha)$ contribution and $C_7^P(M_W)$ is proportional to s_W^2, we need to consider in eq. (2.3) only the part of $C_7^P(M_W)$ enhanced by M_t^2 (which actually approximates the full Wilson coefficient very well [19]).

Our results for the $O(g^2)$ correction to the Wilson coefficients $C_{7,8}^{\rm eff}(M_W)$ are accurately approximated by

$$\delta C_{7,\rm ew}^{\rm eff} = \frac{g^2}{16\pi^2}\left[1.8615 - 2.422\left(1-\frac{\overline{M}_t}{175.5}\right) - 0.4463\ln\frac{M_H}{100} - 0.216\ln^2\frac{M_H}{100}\right],$$
$$\delta C_{8,\rm ew}^{\rm eff} = \frac{g^2}{16\pi^2}\left[0.2596 + 0.282\left(1-\frac{\overline{M}_t}{175.5}\right) - 0.1366\ln\frac{M_H}{100} - 0.021\ln^2\frac{M_H}{100}\right], \quad (2.5)$$

where the $\mathrm{SU}(2)_L$ coupling g can be calculated from the relation $g^2 = 4\sqrt{2}G_\mu M_W^2$. We use $M_W = 80.419\,\mathrm{GeV}$ and $s_W^2 = 0.23145$ for the $O(g^2 s_W^2 M_t^2)$ contributions that we retain. Eqs. (2.5) reproduce accurately (within 1%) the analytic results in the ranges $100 < M_H < 250\,\mathrm{GeV}$ and $165 < \overline{M}_t < 180\,\mathrm{GeV}$. We stress that eqs. (2.5) are independent of the choice of the scale μ_t in the QCD top mass definition: it is sufficient to calculate $\overline{M}_t(\mu_t)$ and employ it in eqs. (2.5). Differences between different choices are present in the QCD corrections to BR_γ but are higher order effects as far as the present calculation is concerned.

The numerical relevance of our corrections to $C_{7,8}^{\rm eff}(M_W)$ is shown in figure 2 for $\overline{M}_t = 175.5\,\mathrm{GeV}$: at $M_H = 100\,\mathrm{GeV}$ the Wilson coefficients of $Q_{7,8}$ are reduced, respectively by 2.6% and 0.7%. As a measure of the uncertainty due to the expansion around $s_W = 0$ we use the difference between the complete correction to the muon decay and its $s_W \to 0$ limit, which amounts to about 0.5%. This seems to us a realistic estimate of the error due to our approximation and we will use it in the following. If we consider only fermionic loops we reproduce the results of [8] for C_7, which lead to a -2.3% reduction of $C_7(M_W)$. Although purely accidental, the closeness of this fermion loop approximation to our complete result for a light Higgs, $M_H \approx 100\,\mathrm{GeV}$ is impressive.

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3. Heavy Top Expansion and the gaugeless limit

Let us now consider the HTE of our results and see how it compares with existing analyses. In units $g^2/(16\pi^2)\,\overline{M}_t^2/(2M_W^2)$ it is given by

$$
\begin{aligned}
\delta C^{\rm eff,ew}_{7,\rm HTE} = {} & \frac{55h_t - 16 - 11h_t^2 - 26h_t^3}{144h_t} - \frac{2h_t - 16 - 36h_t^2 + 74h_t^4 - 45h_t^5 + 2h_t^6}{864h_t^2}\pi^2 + \\
& + \frac{s_W^2}{27} - \frac{8 - h_t - 6h_t^2 - 52h_t^3 + 85h_t^4 - 33h_t^5 + 2h_t^6}{72h_t^2}\,\mathrm{Li}_2(1-h_t) - \\
& - \frac{74 - 45h_t + 2h_t^2}{288} h_t^2 \ln^2 h_t - \\
& - \frac{80 + 68h_t - 262h_t^2 + 134h_t^3 - 25h_t^4 + 2h_t^5}{288h_t}\,\phi\left(\frac{h_t}{4}\right) + \\
& + \frac{8 - 17h_t - 2h_t^2 - 14h_t^3}{72h_t}\ln h_t
\end{aligned}
\tag{3.1}
$$

and

$$
\begin{aligned}
\delta C^{\rm eff,ew}_{8,\rm HTE} = {} & \frac{32 - 83\,h_t - 23\,h_t^2 + 16\,h_t^3}{96\,h_t} - \frac{8 - h_t - 18\,h_t^2 - h_t^4 + 9\,h_t^5 - h_t^6}{144\,h_t^2}\pi^2 - \frac{s_W^2}{9} + \\
& + \frac{16 - 2h_t - 12h_t^2 + 40h_t^3 - h_t^4 - 30h_t^5 + 4h_t^6}{48\,h_t^2}\mathrm{Li}_2(1-h_t) + \\
& + \frac{1 - 9h_t + h_t^2}{48} h_t^2 \ln^2 h_t + \\
& + \frac{8 - 58\,h_t + 62\,h_t^2 + 17\,h_t^3 - 16\,h_t^4 + 2\,h_t^5}{96\,h_t}\,\phi\left(\frac{h_t}{4}\right) - \\
& - \frac{8 + h_t + 7h_t^2 - 5h_t^3}{24\,h_t}\ln h_t\,,
\end{aligned}
\tag{3.2}
$$

where $h_t = M_H^2/\overline{M}_t^2$, $\mathrm{Li}_2(x)$ is the dilogarithmic function, and $\phi(x)$ is given in [17, eq. (48)]. As can be seen in figure 2, the leading HTE term approximates our full result very poorly, especially for a light Higgs. We have also studied the convergence of the HTE, calculating its first three terms, and found that for realistic M_t values they do not converge, in a way very similar to [17]. Our eq. (3.1) differs from the analogous one in [11] by a term $\frac{2}{9}\left(\frac{1}{2} - \frac{s_W^2}{3}\right) + \frac{s_W^2}{9}$. On the other hand, we agree with [11] if we perform the calculation in the *gaugeless* limit. This is not surprising because it is known [18] that the *gaugeless* approximation does not always include all leading M_t^2 contributions.

To understand better this point, notice that for a asymptotically heavy top both the top Yukawa coupling, $g_t = g\,M_t/(2M_W)$, and the loop integration can provide powers of the top mass. In the case at hand, the one-loop integrals are convergent, so that the one-loop contributions scale at most like $g_t^2/M_t^2 \sim g^2/M_W^2$. At the two-loop level, the *gaugeless* contributions scale as $g_t^4/M_t^2 \sim g_t^2\; g^2/M_W^2 \sim g^4 M_t^2/M_W^2$, but the

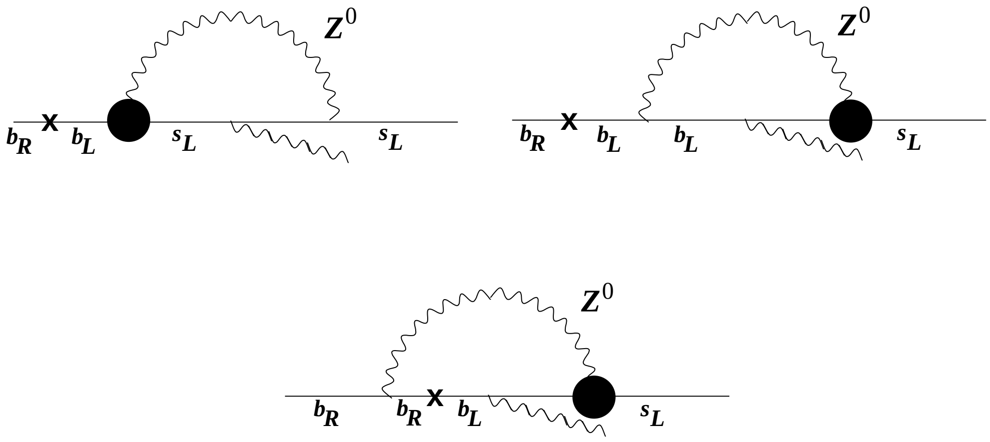

Figure 3: Insertions of an effective Z^0 penguin vertex in a one-loop diagram.

same heavy top behaviour can be obtained by inserting a dimension four operator[1] proportional to g_t^2 in a topless loop. In general, the effective lagrangian obtained after integrating out the heavy top tells us exactly which the relevant operators are [31]. In our case, only the diagrams in figure 3 contribute to the leading HTE through the insertion of a flavor changing dimension four Z^0 penguin operator of the kind $\bar{s}_L \gamma^\mu b_L Z^0_\mu$. The diagram with a mass insertion on the internal b line depends on the regularization scheme — it vanishes if IR divergences are regulated dimensionally — and in the schemes where it does not vanish it is cancelled in the matching by a contribution from the electroweak penguin operator Q_7^P of eq. (2.4). In both cases, however, its contribution is reintroduced in the quantity $C_{7,8}^{\rm eff}$ by C_7^P. In the limit of a heavy top the effective vertex has the form

$$\Gamma^\mu_{\bar{s}bZ^0} = i \frac{g^3}{16\pi^2} \frac{\lambda_t}{c_W} \frac{x_t}{4} \, \bar{s}_L \gamma^\mu b_L \tag{3.3}$$

with $c_W = \cos\theta_W$ and $\lambda_t = V_{ts}^* V_{tb}$. Inserting this gauge-independent effective coupling in the one-loop diagrams of figure 3, and keeping in mind the tree level couplings of the Z^0 boson with b_L, $\sim (1/2 - s_W^2/3)$, and with b_R, $\sim -s_W^2/3$, we obtain the difference between the HTE of our result and the *gaugeless* limit. The argument is completely analogous for C_8, whose HTE also differs from the *gaugeless* approximation.

So when does the *gaugeless* limit potentially fail at two-loop? Whenever at the one-loop level the top quark diagrams in the limit of a heavy top scale like a constant, namely in the same way as the topless contributions. Indeed, in this case we know that there are some dimension four effective operators proportional to g_t^2 that can be inserted in one-loop diagrams not containing the top and give contributions of the same order, in the limit of heavy top, of those belonging to the two-loop *gaugeless* approximation.

[1] Dimension two insertions are removed by our choice of renormalization in the $W - \phi$ sector.

	$bs\gamma, bsg$	$H\gamma\gamma$, Hgg [18, 32]	$\Delta\rho, R_b, HZZ, K\to\pi\nu\bar{\nu}$ [34, 18, 33]	B_0-$\bar{B}_0$ [17]
One loop	$\frac{g_t^2}{M_t^2} \sim \frac{g^2}{M_W^2}$	$\frac{(e^2,g_s^2)\,v\,g_t M_t}{M_t^2} \sim (e^2, g_s^2)$	$\frac{g^2 M_t^2}{M_W^2}$ or g_t^2	$\frac{g_t^4}{M_t^2}$
Two loop	$\frac{g_t^4}{M_t^2} \sim \frac{g^2 g_t^2}{M_W^2}$	$\frac{(e^2,g_s^2)\,v\,g_t^3 M_t}{M_t^2} \sim (e^2, g_s^2)\, g_t^2$	$\frac{g^2 g_t^2 M_t^2}{M_W^2}$ or g_t^4	$\frac{g_t^6}{M_t^2}$

Table 1: Leading HTE contributions to different processes. Following the criterion given in the text, the *gaugeless* limit fails in the first two cases.

Table 1 summarizes the situation for the processes considered in the literature at the two-loop level. It should be clear that the *gaugeless* limit works safely only when the asymptotic expansion in M_t has maximal power (M_t^2 at one-loop, M_t^4 at two loop). Of course, there might be exceptions. Indeed, whether the $O(g_t^2)$ dimension four operators are relevant or not depends on the process under consideration. For instance, in the Hgg effective vertex [32] — relevant for gluon-gluon fusion production and hadronic decays of the SM Higgs boson — they are not because the gluons have no electroweak interaction. This is in contrast to the similar case of the $H\gamma\gamma$ effective vertex [18], where the *gaugeless* approximation does not give the correct result. Similar considerations apply to the heavy Higgs limit, although the leading term in the heavy Higgs expansion, subject to other constraints, is not always what is expected from dimensional analysis.

4. Effects on the branching ratio

Let us now examine in some detail the effect of our calculation on BR_γ. As a first step we calculate the Wilson coefficients at a scale $\mu_b = 4.8\,\mathrm{GeV}$. It is well known that the large mixing between $Q_2 = (\bar{c}b)_{V-A}\,(\bar{c}s)_{V-A}$ and Q_7 induces additive terms in the running of the coefficients from the W to the b mass scale which are numerically very important. Our aim is to resum all contributions $O(g^2\alpha_s^n L^n)$ and $O(\alpha M_t^2/M_W^2 \alpha_s^n L^n)$, where L is a large logarithm. As mentioned above, these terms are uniquely originated by heavy degrees of freedom and enter only the determination of the Wilson coefficients at a high scale $\mu \approx M_W$. At this order the evolution of the coefficients is therefore driven only by LO QCD effects. The Wilson coefficient at the bottom mass scale is given at LO in QCD by

$$C_7^{(0)\mathrm{eff}}(\mu_b) = \eta^{16/23} C_7^{(0)\mathrm{eff}}(M_W) + \frac{8}{3}\left(\eta^{14/23} - \eta^{16/23}\right) C_8^{(0)\mathrm{eff}}(M_W) + + C_2^{(0)}(M_W) \sum_{i=1}^{8} h_i \eta^{a_i}\,, \tag{4.1}$$

where $\eta = \alpha_s(\mu_W)/\alpha_s(\mu_b) \approx 0.56$ and h_i, a_i are constants given e.g. in [30]. The last term is approximately equal to -0.173 and is dominant.

i	$P7$	$P8$	$P9$	$P10$
$\hat{\gamma}^{(0)\mathrm{eff}}_{s,i7}$	$-\frac{16}{9}$	$-\frac{1196}{81}$	$\frac{232}{81}$	$\frac{1180}{81}$
$\hat{\gamma}^{(0)\mathrm{eff}}_{s,i8}$	$\frac{5}{6}$	$-\frac{11}{54}$	$-\frac{59}{54}$	$-\frac{46}{27}$

Table 2: Anomalous dimension matrix entries relevant for the mixing between electroweak and magnetic penguins [10] in the basis of [30]. Two *up* and three *down* active flavors are assumed.

The electroweak corrections affect eq. (4.1) in two ways: (i) they shift $C_i^{(0)\mathrm{eff}}(M_W)$ by $\delta C^{\mathrm{eff}}_{i,\mathrm{ew}}$; (ii) they introduce in eq. (4.1) those gluon and electroweak penguin operators that have non-zero $O(g^2)$ or $O(\alpha M_t^2)$ contributions to the Wilson coefficients at $\mu = M_W$. In the basis of [30], these are $Q^P_{3,7,9}$. Their LO QCD mixing with the magnetic penguin operators can be gleaned from the anomalous dimension matrix $\hat{\gamma}^{(0)\mathrm{eff}}_s$ given in [10, 35] after a change of basis. The new entries of $\hat{\gamma}^{(0)\mathrm{eff}}_s$ calculated in [10] are given in table 2 in the conventional basis adopted in [30]. The additional contributions to eq. (4.1) are therefore

$$\delta^{\mathrm{ew}} C_7^{(0)\mathrm{eff}}(\mu_b) = \eta^{16/23}\delta C^{\mathrm{eff}}_{7,\mathrm{ew}} + \frac{8}{3}\left(\eta^{14/23} - \eta^{16/23}\right)\delta C^{\mathrm{eff}}_{8,\mathrm{ew}} + \\ + \sum_{i=3,7,9} C_i^P(M_W) \sum_j h_{ij}\eta^{a_{ij}} \,, \tag{4.2}$$

where $C_i^P(M_W)$ are the relevant $\mathrm{O}(\alpha)$ contributions to the Wilson coefficients and h_{ij}, a_{ij} are magic numbers that can be easily determined from the anomalous dimension matrix. The last term in eq. (4.2) is approximately given by $0.15\, C_3(M_W) + 0.12\, C_7(M_W) - 0.03\, C_9(M_W)$ and is numerically very small; it reduces $C_7^{(0)\mathrm{eff}}(\mu_b)$ by -0.2%.

Notice now that $C_2^{(0)}(M_W)$ in eq. (4.1) is unaffected by electroweak corrections of the kind considered here if G_μ is used to normalize the effective hamiltonian (as in fact we do); in that case $C_2^{(0)}(M_W)$ does however receive $O(g^2 s_W^2)$ corrections. In the NDR scheme we find

$$C_2(M_W) = 1 + \frac{\alpha(M_W)}{4\pi}\left[-\frac{22}{9} + \frac{4}{3}\ln\frac{M_Z^2}{M_W^2}\right] + O(\alpha_s)\,, \tag{4.3}$$

where $\alpha(M_W)$ is the electromagnetic running coupling evaluated at M_W. The $\mathrm{O}(\alpha)$ electroweak corrections to the Wilson coefficient C_2 are therefore very small (-0.13%). But we stress that a different choice of normalization would induce additional (much larger) electroweak contributions, as can be easily seen using [17]. Eq. (4.3) is a new result that improves on [36], where only QED effects were taken into account, and includes all one-loop electroweak contributions.

From eq. (4.1) and neglecting the $\mathrm{O}(\alpha)$ effects of eq. (4.3) we see that $C_7^{(0)\mathrm{eff}}(\mu_b)$ is reduced by only $(1.3 \pm 0.2)\%$ for $M_H = 100\,\mathrm{GeV}$ due to the electroweak corrections we have calculated; the reduction is less pronounced for larger Higgs masses.

In a similar way, we find that for $C_8^{(0)\rm eff}(\mu_b)$ the numerical impact of the last term in eq. (4.2) is more important than that of the two-loop correction; electroweak effects increase $C_8^{(0)\rm eff}(\mu_b)$ by about $0.3 \pm 0.2\%$ for $M_H = 100\,\mathrm{GeV}$. In addition to the leading logarithmic QCD effects considered in eq. (4.1) there are next-to-leading QCD terms that enter the calculation of BR_γ [3]. Our electroweak corrections affect them only as a NNLO effect. As there are other uncalculated contributions to that order, even in the approximation we have adopted — in particular $O(g^2\alpha_s)$ corrections to the Wilson coefficients, as can be seen from [19, eq. (3.14)] — our corrections should be implemented only in the calculation of the LO QCD Wilson coefficient $C_7^{(0)\rm eff}(\mu_b)$.

The total effect of the electroweak corrections we have calculated on the NLO calculation of BR_γ is a $-2.0 \pm 0.3\%$ reduction for a light Higgs mass, $M_H \approx 100\,\mathrm{GeV}$. We stress that this does not include the $O(\alpha\alpha_s^{n-1}\ln^n m_b/M_W)$ QED effects of [8, 9, 10]. For larger Higgs masses the effect becomes smaller: -1.6% for $M_H = 200\,\mathrm{GeV}$ and -1.3% for $M_H = 300\,\mathrm{GeV}$. In the *gaugeless* approximation and excluding the last contributions to eq. (4.2), the net effect on BR_γ is a 0.5% reduction for $M_H \approx 100\,\mathrm{GeV}$.

We now calculate BR_γ following closely [3, 5, 8, 9] and including all known higher order effects as well as our new results. We also update the input parameters and introduce some minor refinements in the NLO analysis. For instance, we evaluate the $\overline{\mathrm{MS}}$ top mass from the pole top mass using the $O(\alpha_s^2)$ expression [37], as these corrections are large and their origin is distinct. Compared to the use of the $O(\alpha_s)$ conversion formula, this leads to a -0.4% reduction of BR_γ. With respect to the detailed analysis of [5], we adopt a new CKM factor $f_{\rm CKM} = |V_{ts}^* V_{tb}/V_{cb}|^2 = 0.97 \pm 0.02$ instead of 0.95 ± 0.03, obtained using $0 < \bar\varrho < 0.4$ from global fits of the unitarity triangle [38]. For the semileptonic BR, we employ $\mathrm{BR}_{\rm SL} = 0.1045 \pm 0.0021$, corresponding to the Υ resonance determination (the average of LEP measurements is 0.1073 ± 0.0018). We also use $M_t \equiv M_t^{\rm pole} = (174.3 \pm 5.1)\,\mathrm{GeV}$, $\alpha_s(M_Z) = 0.119 \pm 0.002$, $M_W = 80.419\,\mathrm{GeV}$ [27, 16], $\lambda_2 = 0.12\,\mathrm{GeV}^2$, $r_{cb} = m_c/m_b = 0.29 \pm 0.02$, $M_{cb} = m_b - m_c = 3.39 \pm 0.04\,\mathrm{GeV}$ [39]. The above value of r_{cb} is equivalent to a pole mass $m_b = 4.77 \pm 0.15\,\mathrm{GeV}$. Employing a conventional definition of *total* BR_γ [9] with a cut on the photon energy $E_\gamma > (1-\delta)\,m_b/2$, $\delta = 0.9$, we obtain

$$\mathrm{BR}_\gamma = 0.000329 \frac{f_{\rm CKM}}{0.97} \frac{\mathrm{BR}_{\rm SL}}{0.1045} \left(\frac{\alpha_s(M_Z)}{0.119}\right)^{1.13} \left(\frac{M_t}{174.3}\right)^{0.48} \left(\frac{r_{cb}}{0.29}\right)^{0.68} \left(\frac{M_{cb}}{3.39}\right)^{-0.3}. \quad (4.4)$$

Here the dependence of the calculation on the main input parameters is summarized for small ($< 1\,\sigma$) variations around their central values. Notice that, compared to [9, 40], the 2% reduction due to electroweak corrections is compensated by a 2% increase from the update of the input parameters. The largest present single parametric uncertainty comes from r_{cb} and reaches around 5%.

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The choice of $\delta = 0.9$ for the photon cut-off energy in eq. (4.4) is mainly motivated by the need to compare with previous literature. The experimental measurement is based on a much stronger cut, $E_\gamma > 2.1\,\mathrm{GeV}$ [1], but needs to be extrapolated to a more inclusive branching fraction (see [9] for a recent discussion). On the other hand, non-perturbative problems may arise for soft photons. It seems therefore useful to know BR_γ for higher and more realistic cut-offs. For $0.3 < \delta < 0.9$ the central value of eq. (4.4) is very well approximated by

$$\mathrm{BR}_\gamma(\delta) = 3.01 + 1.01\,\delta - 1.49\,\delta^2 + 0.79\,\delta^3\,, \tag{4.5}$$

which shows a mild dependence on the cut-off in a large region of δ [9].

5. Summary

In summary, we have reanalyzed in detail the two-loop electroweak corrections to $B \to X_s\gamma$ and $B \to X_s g$ decays. In order to avoid dealing with presumably small photonic effects, in our calculation we have neglected terms proportional to s_W^2 not enhanced by M_t^2. We have also accurately discussed the interplay between electroweak and QCD corrections. As a byproduct of the calculation we have presented the complete $\mathrm{O}(\alpha)$ corrections to the Wilson coefficient $C_2(M_W)$. The total effect of electroweak corrections on the BR of $B \to X_s\gamma$, BR_γ, is a $-2.0 \pm 0.3\%$ reduction, which is three times larger than in [11]. After inclusion of all NLO QCD contributions, of non-pertubative corrections and of all known QED and electroweak effects, we find (for $M_H = 100\,\mathrm{GeV}$ and $\delta = 0.9$)

$$\mathrm{BR}_\gamma = (3.29 \pm 0.21 \pm 0.21) \times 10^{-4}\,, \tag{5.1}$$

where the first error is parametric and based on up-to-date experimental inputs and the second one is obtained by scanning the various scales considered in [5] between 1/2 and twice their central values and adding 0.3% for unaccounted electroweak higher orders. Had we combined the scale ambiguities in quadrature, the second error would have been 0.16. For $M_H = 215\,\mathrm{GeV}$, at the other edge of the preferred region from global fits, the central value of BR_γ is 3.30×10^{-4}. Eq. (5.1) is in good agreement with the present experimental values $\mathrm{BR}_\gamma = (3.14 \pm 0.48) \times 10^{-4}$ from Cleo and $\mathrm{BR}_\gamma = (3.34 \pm 0.68) \times 10^{-4}$ from Belle [1].

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